

I. Introduction.

Saint Venant's mathematical solution for shearing stresses in bars subjected to torsion and bending was successfully applied only in a few of those special cases where the boundary line of the cross section is comparatively simple. Using Saint Venant's method as a basis, it can be proven that both torsion and bending problems can be solved by using soap films.² This paper describes the equipment developed for making soap film tests, and it also gives several results obtained. In the case of torsion, one simple problem is given in order to describe the method of establishing the degree of precision of the soap film procedure. The problem of twist for angles having various fillet radii is developed and the stress curve is given in Figure 9. In the case of bending, circular, rectangular, and I-beam sections are chosen for soap film analysis. The circular and rectangular results are in good agreement with the Saint Venant theory. I-beam results show stresses of the same character as indicated by elementary theory.

II. Torsion Problem.

Saint Venant's mathematical solution for shearing stresses in bars subjected to torsion reduces to solution of the equation

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = -2G\theta \quad (1)$$

ϕ is a stress function which is constant on the boundary of the cross section of the bar. x and y are coordinates of the section. G is the shearing modulus. θ is the angle of twist per unit length.

It can be shown that the same equation applies for a loaded membrane stretched over a hole in a flat plate, the boundary outline of the hole being the shape of the cross section of the twisted bar for which the stresses are desired. Such a membrane is illustrated in Figure 1.

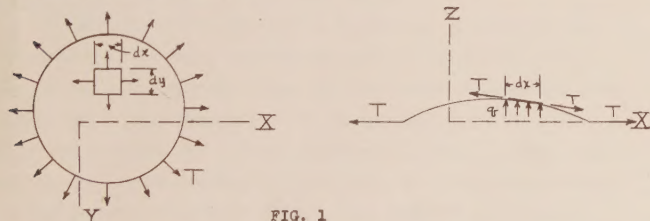


FIG. 1

The load is a uniform pressure q per unit of area, exerted from below. There is a tensile force T per unit length of boundary. The equation of the membrane for small deflections is

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = -\frac{q}{T} \quad (2)$$

By comparing this with equation (1) it is seen that a relationship can be established between the vertical displacements z of the soap film and the stress function ϕ . From the deflection of the membrane the stress function can be obtained by the ratio $\phi/z = 2G\theta (q/T)$.

It is shown in several treatises on elasticity that the shearing stress τ_{xz} Figure 2, in the x direction at any point in the cross section xy of the twisted bar can be found by measuring the

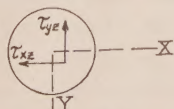


FIG. 2

¹From a dissertation presented for the degree of Doctor of Science at the University of Michigan.

²This method of solving torsion problems was proposed by L. Prandtl. For bending the method was applied by Griffith and Taylor, British Advisory Committee for Aeronautics, Vol. 3, 1917.

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NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society

slope $\partial z/\partial y$ of the corresponding soap film and then substituting in the equation

$$\tau_{xz} = 2G\theta \frac{T}{q} \frac{\partial z}{\partial y} \quad (3)$$

In a similar manner the shearing stress in the y direction is

$$\tau_{yz} = 2G\theta \frac{T}{q} \frac{\partial z}{\partial x} \quad (4)$$

The ratio T/q may be eliminated in testing by stretching soap films on two holes, one circular and the other of the same shape as the cross section of the shaft under investigation. The ratio of the slopes gives directly the ratio of stresses in the two shafts, for the same angle of twist.

Figure 3 shows a test plate erected in position for testing. The circular hole provides a known standard for comparison with the 5" x 5" x 1" angle. The radius of the circular hole is $r = 2A/p$ where A is the area of the test hole and p is its perimeter. See mathematical derivation in Appendix I.

Two methods for finding the slopes of soap films are available, namely the contour lines method and the light reflection method.

Contour Lines Method. Figure 4(a) shows a contour lines map taken from a soap film stretched over a square hole. The numbers at the left indicate the elevations of the respective contour lines. Horizontal distances between contour lines measured from each of the four points a in Figure 4(a) are tabulated in Figure 4(b), together with the vertical distance differentials. By plotting horizontal and vertical distances given in this table there is obtained an enlarged section of the contour map at any point a along a plane perpendicular to the boundary line as given in Figure 4(c). Such a plot always shows the greatest film slope at the boundary, which indicates that stress is maximum at the boundary. The angle at the boundary may be determined with sufficient accuracy by drawing a tangent line to the film curve of Figure 4(c). By interpolation of sufficient independent readings a very good accuracy may be obtained in most cases.

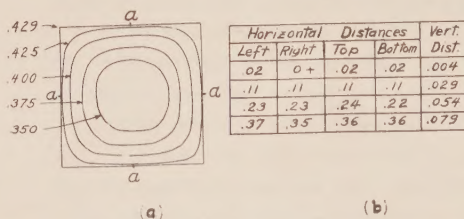


FIG. 4

Light Reflection Method. Figure 5 illustrates a second method for finding the slope of a soap film at its boundary. A light is so

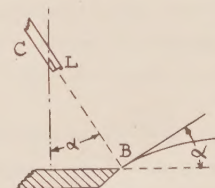


FIG. 5

placed on the end L of a telescope C that the rays of light pass down to the film at the boundary B and are reflected back over the same path to the telescope. A protractor rigidly attached to the telescope gives angle α between the light path and the vertical. This α is equal to the desired α which the tangent to the film at the boundary makes with the horizontal. This method gives results with speed and facility. It is sometimes not as accurate as the contour method. It always gives angles which are too small, and under certain conditions it fails altogether.



FIG. 6

Figure 6 indicates the reason for errors inherent in the light reflection method of angle measurement. This illustration consists of the film section given in Figure 4(c) with the addition of a dotted line to indicate a very slight overlapping of the film onto the plate surface at the boundary. When the boundary line is straight or convex the overlapping is extremely small and may not lessen the angle measurement much over five percent. When the boundary line is sharply concave, the overlapping will be of the relative magnitude shown in Figure 6 and may extend .02" or .03" above the elevation of the flat plate. This is not a sufficient amount to change any contour points. It is enough to prevent reasonably accurate measurements with the reflection telescopic instrument.

Equipment. Figure 7 shows the equipment used for contour recording and the light reflection method of angle measurement. The upper and lower cast iron boxes hold a flat test plate made of aluminum at least 1/16" thick. The lower box is nearly full of water, the height of which may be regulated by moving the reservoir with rubber hose connection along the inclined plane at the right. A film is drawn over the test hole in the aluminum plate by means of a celluloid spreader with wire handle attached. The spreader is placed within the upper iron box, the handle passing out through a hole in the side of the box. Motion of the reservoir up the inclined plane produces a pressure underneath the film, thus giving it a deflection. The amount of deflection should be such as to make angles at the boundary of 10 to 40 degrees unless some special reason requires an angle outside this range.

A glass plate with micrometer depth gauge attached through its center may be slid to any desired position in its horizontal plane on the upper iron box. This provides means of measuring film heights, while the swinging contour record board facilitates recording of the position of each contact as made on the soap film. The upper pointer of the micrometer depth gauge is at the proper elevation and suitably sharpened to make a fine impression on a paper placed on the contour record board.

It is not necessary to lift the glass plate at any time. A glass funnel is used to pour soap film solution onto the test plate, a second small hole in the side of the box being provided for this purpose. The solution used consists of 2 grams of sodium oleate, 30 cubic centimeters of glycerine, and 1 liter of distilled water. Addition of more glycerine or sodium oleate is perhaps desirable when the test holes are more than one inch in width. By using precautions against vibration and atmospheric disturbance it is possible to maintain films for twenty-five hours.

The angle measuring apparatus shown in Figure 7 consists of a surveyor's plane table telescope with protractor, an electric lamp, and a supporting frame conveniently constructed for shifting the plane and elevation of the lamp. Just above the tripod base there is a ball bearing and a swinging arm which revolves on the ball bearing.

The swinging arm carries a vertical 2" pipe, machined to provide close vertical sliding contact for a telescope supporting frame. The whole sliding part is balanced on a pulley by a lead weight inside of the 2" pipe.

In operation, the telescope is brought into the vertical plane of the film angle which it is desired to measure, and the lower ball bearing is clamped. The lamp switch is closed. The telescope is focused on the boundary of the film at the point under measurement, and the cross hairs and light reflection brought into clear view.

Then the telescope is lowered until the spot or streak of light half disappears at the boundary of the film. The angle is read and the height of film is taken by means of micrometer readings at the crest of the film. The several motions can be readily acquired with a little practice. The accuracy of the observer's readings can be checked by making readings of angles on a circular boundary and then computing the angle from the micrometer readings at crest and base plate and the known radius of the circular hole. In testing, a table was computed for quick calculation, giving angles at the boundary versus deflection of film above the test plate, for any circular section used.

Determination of Accuracy. In order to determine the accuracy of stress results found by soap film measurement, a series of contour tests was made on a circle of 1.51" diameter and a 1.51" x 1.51" square, both used on the same test plate. The ratio of the maximum stress at the boundary of the square to the stress at the boundary of the circular section was determined.

The stress at the side of a square is theoretically $G\theta b c/\alpha$, and the stress at the boundary of a circular section is $G\theta d/2$ where Saint Venant gives values for the factors $\beta = .141$, and $\alpha = .208$. c is the dimension of the side of the square, 1.51", and d is the diameter of the circular hole, 1.51". G , the shearing modulus, and θ , the angle of twist per unit length, will cancel upon taking the stress ratio for square to circular section.

$$\text{Stress ratio} = \frac{G\theta b c/\alpha}{G\theta d/2} = \frac{2\beta c}{\alpha d} = \frac{2 \times .141 \times 1.51}{.208 \times 1.51} = 1.355$$

When film deflections are small the soap film method commonly gives within 2% of this value. Only a very few scattering measurements will be as much as 4% in error. Since this stress ratio is known to be accurate, it might be concluded that 2% is a possible accuracy for the contour method in general, provided enough measurements are taken to furnish suitable data for interpolation. This conclusion seems to hold except in the case of small reentrant corners, which is discussed in the next article.

Stress at a Reentrant Corner for Twisted Angles. As an example of more complicated problems, the soap film method was applied to find the stress existing at the reentrant corner of four angle sections in torsion. The dimensions of the sections were 5" x 5" x 1" with radii at the reentrant angle of respectively 1", 1/2", 1/4", and 1/8". Figure 8 shows the contour lines resulting from the test when the reentrant radius was 1/4". The film slopes were determined at the reentrant angle and at the middle of the long side, using the contour method of angle measurement.

In the middle curve of Figure 9 these stress ratios are given.

It is probable that the contour lines do not give satisfactory accuracy at the reentrant corner when the radius of the fillet is small. The actual film slope is changing rapidly at such points, and its magnitude is larger than can be obtained by the contour lines method.

The light reflection method is not satisfactory in this case by virtue of the phenomenon illustrated in Figure 6. Using the light reflection method described in connection with Figure 5, we checked the results of Griffith and Taylor as shown in the lower curve. It is seen that the factors of stress concentration obtained in this way are lower than the correct values.

For comparison there is also given a curve of stress ratios obtained by using the equation due to Trefftz which is

$$\text{Stress ratio} = 1.74 (c/r)^{1/3} \quad (5)$$

r is the reentrant radius in inches. c is the width of legs of angle in inches, which is unity for the sections tested. The Trefftz equation was specifically intended for small radii of fillet.

It has been observed in experimental summaries that there exists a greater diversity of occasional measurements in the case of the reentrant angle at small radii. It may be said in explanation that this is the most difficult problem for contour mapping, and that it is probably not susceptible to measurement with a precision as good as 2% where the radius is as small as 1/4". 4% is safer estimate of precision in such a case, and the accuracy is much less for a reentrant radius of 1/8".

III. Bending Problem

In the case of the bending of a prismatical bar, Figure 10, the shearing stress components can be obtained from a stress function satisfying the equation

$$\frac{\partial^2 \phi}{\partial x^2} - \frac{\partial^2 \phi}{\partial y^2} = 0 \quad (6)$$

and having on the boundary values determined by the equation

$$\phi = \left[\frac{1}{2} \int x^2 dy - \frac{y^3}{26} \right] P/I \quad (7)$$

See Appendix II. Equation (6) is the equation of an unloaded membrane.

The stress distribution can be obtained by using a soap film stretched on a boundary of which the heights are determined by equation (7).

x is the vertical axis through the beam section.

y is the horizontal axis through the beam section.

P is the applied load.

I is the moment of inertia of the section.

Poisson's ratio was taken as 0.3, a satisfactory value for a steel beam.

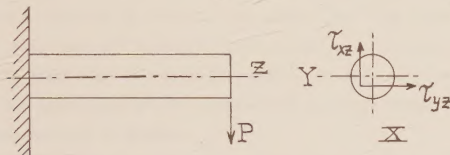


FIG. 10

Thus the problem of experimental determination of shearing stresses reduces to building a special supporting model such that the boundary line will have the heights calculated from equation (7). These models are satisfactorily built from thin annealed sheet brass. Figure 11 is such a model made for a circular beam of 3" diameter, with certain reduced displacements to be explained in the next paragraph. The horizontal projection of the boundary of the opening is the exact beam section. The soap film stretched on the boundary surface would give the correct value of ϕ , as the elevation at any point of the coordinates (x,y) that may lay within the section.

When values of ϕ are computed, from equation (7), around the boundary of any cross section of structural shape, it will be found that the resulting boundary line has such excessive variations of elevations that film drawing and mapping on the constructed model would be inconvenient, and would also fail to satisfy the requirement of small membrane slopes. To remove this difficulty we introduce another function ϕ' , by superposing on ϕ a linear function. Then

$$\phi' = \frac{1}{2} \int x^2 dy - \frac{y^3}{26} - Bx - Cy + D$$

where B and C are constants that are arbitrarily chosen by trial so as to give ϕ' the desired range of elevations along the boundary line. The ratio P/I is also dropped at this place, and reintroduced later, so as to simplify calculations. Certain methods of selecting B and C become apparent after practical application to a section. B may become zero in special cases, both in symmetrical and unsymmetrical sections. Generally C will not be zero. D is a vertical height such as to make the vertical supporting walls under the boundary surface strong mechanically. In addition, accuracy of measurement requires that D be made as large as will permit of placing the model in the test box with the shortest possible micrometer shaft in use. The short micrometer shafts are preferable from the standpoint of reducing the eccentricity of the shaft and of improving visibility.

After the above process has been carried out to its greatest degree of slope reduction, it will often be necessary to still further reduce the range of variation in elevation. This can be accomplished by the introduction of a multiplying factor which gives the boundary elevations for the brass model as:

$$z = \phi'/k \quad (9)$$

It may be desirable in addition to use an enlarged scale such as double size for all x and y dimensions of the cross section. This scale enlargement does not alter the z dimensions at all, and its effect is taken care of as a multiplying factor, S, which will appear in the expressions for shearing stress.

Mathematical deductions in the appendix show that, having the contour lines of the soap film, the stress components of the section can be calculated as follows:

$$\tau_{xz} = (SPk/I) \partial z / \partial y - Px^2/2I + 3y^2P/26I + CP/I \quad (10)$$

$$\tau_{yz} = -(SPk/I) \partial z / \partial x - BP/I \quad (11)$$

where S is the ratio of linear scale increase,

k, B, C are constants noted above,

x and y are the coordinates on the actual cross section.

Circular Cross Section in Bending. In order to obtain a measure of accuracy of the soap film method in bending, the case of a circular cross section for which the exact solution of the problem is known was considered first. The values of boundary elevations were computed for a circular section of 3" diameter by combining equations (7) and (8). The coordinates of the circle were taken as in Figure 12. The equation of the circle is $x^2 + y^2 - r^2 = 0$ where $r = 1.5$ ". It may be readily shown that the first term of equation (8) is

$$\frac{1}{2} \int_{y_n}^{y_{n+1}} x^2 dy = 1.125(y_{n+1} - y_n) - .1667(y_{n+1}^3 - y_n^3) \quad (12)$$

when taken from any point n to the next point n+1. This expression may be computed as a differential from each point to its succeeding point. The value of the term $1/2 \int x^2 dy$ at any point is the sum of all of the differential terms of equation (12) preceding the point. As a check on this calculation it is well to draw a graph of x^2 against y as given in the middle of Figure 12. The summation of areas up to any point gives the desired $\int x^2 dy$.

This work is arranged in part in a table in Figure 13.

ϕ is obtained from equation (7) neglecting P/I. In this case full scale is used and k=1 in equation (9). ϕ' is obtained from ϕ by adding a -Cy term where C = .7788. This value of C was so taken as to make the range of boundary elevations ϕ' a minimum. The right hand illustration of Figure 12 shows the values of both ϕ and ϕ' . The latter quantity gives the boundaries used in constructing the model shown in Figure 11.

Circular Section in Bending
Abbreviated calculation of $\phi' = \frac{x^2 dy}{2} - \frac{y^3}{26} - .7788y$

No.	y	x	$\frac{x^2 dy}{2}$	$-y^3/26$	ϕ	$z = \phi'$
0	-1.5	0	0	.1298	.1298	1.298
1	-1.45	-.3840	.00185	.1173	.1191	1.248
3	-1.25	-.8292	.04427	.0751	.1194	1.093
5	-.75	-1.299	.35156	.0162	.3678	.952
7	-.25	-1.479	.84647	.0006	.8471	1.042
8	0	-1.5	1.125	0	1.125	1.125
11	.75	-1.299	1.8984	-.0162	1.882	1.298
15	1.45	-.3840	2.2482	-.1173	2.131	1.001
16	1.5	0	2.250	-.1298	2.120	.952

FIG. 13

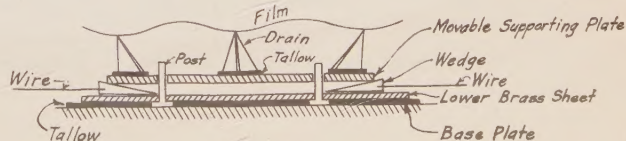


FIG. 14

With all bending films it is necessary to drain the low points of the film. This needs to be done intermittently, or else the film will break from excessive dryness. The circular section model is a good example of drainage problems. There were three low spots on this soap film. Three small pieces of brass, folded over to form a narrow slot in each case, were fixed rigidly on a movable brass plate and the whole mounted under the boundary edge. Two wires with attached wedges were placed under the plate in such a manner that the pushing and pulling of the wires would move the wedges in or out and lift or depress the drains as desired. Two fixed vertical pins passing through holes in the brass supporting plate prevented lateral motion of the drains. The arrangement may be seen in Figure 10 in part, and the idea is indicated by Figure 14.

The contour map for the circular section is given in Figure 15. Inspection will reveal that the film has a zero slope at the center of the section both in the vertical and horizontal directions. The symmetry of the section makes this a very good section for study of the accuracy that can be obtained with the soap film method in bending problems.

Figure 16 indicates the distribution of vertical shearing stresses along the horizontal and vertical diameters of the cross section. The curve in each case gives the exact theoretical values of stress, and the small circles indicate the values derived from experiment with the soap film. The comparison gives a measure of the accuracy that is likely of attainment with soap film experiments in bending. Inspection of the graph for $x = 0$ shows that all soap film values, indicated by circles, are within 2% of the curve. This indicates that all important shearing stresses for the circular section are measurable to within 2% accuracy by the soap film method. The same precision was secured for analysis of a 2" x 4" rectangular section in bending.

The soap film equation (10), for vertical shearing stresses, was used to determine the points of Figure 16. It will be observed that only the first term of equation (10) contains the film slope, and the two additive terms, $3y^2T/26I$ and CP/I , may be of as much influence as the slope term. This fact conduces to good accuracy. The horizontal shearing stresses which are deduced from equation (11) are not as favorably affected, especially as the second term of that equation is commonly zero. However, the horizontal shearing stress is generally small in magnitude and therefore comparatively unimportant.

I-Beam in Bending. By use of the soap film method the resultant stresses all around the boundary of an I-beam subjected to bending were derived. The contour map for the I-beam is shown in Figure 16. The process is to find the vertical and horizontal shearing stresses of equations (10) and (11) by the steps of procedure used in the case of the circle.

The resultant stress at any point is

$$\sqrt{\tau_{xz}^2 + \tau_{yz}^2}$$

At the boundary it acts tangent to or along the line of the boundary. Resultant stress at the boundary is shown graphically in Figure 18.

The curve shows the maximum resultant boundary stresses to occur on the web midway between the flanges of the I-beam. At these boundary points the stress is 30.63 P/I. There is a gradual decrease of shearing stress along the web surfaces from the mid points as the flanges are approached. This is shown in Figure 18 in both (a) and (b). The shearing stress drops off rapidly for points along the reentrant curve under the flange as the web merges into the flange. There is a very small resultant stress varying to a maximum of 0.5P/I at two points on the top of the flange. This is due entirely to the horizontal shearing component.

It is interesting to compute the maximum web stress by elementary theory. The result is 30.2 P/I, as compared with 30.63 P/I by the soap film method. The minimum resultant stress along the web is 25.3 P/I by elementary theory, as compared with 25.8 P/I by the soap film method.

Appendix I

Radius of Circular Hole in Test Plate.

Consider a soap film stretched over a circular hole of radius r and deflected by a load q per unit area. The tension T per unit length at the boundary will be taken to act opposite in direction to the film at the boundary and at angle β with the horizontal flat plate. See Figure 19.

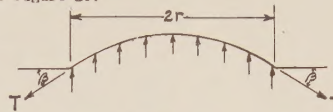


FIG 19

$\pi q r^2$ is the force acting upward on the film. $2\pi T \sin \beta$ is the downward action on the film around the boundary. Equating and solving

$$r = (2T/q) \sin \beta$$

Consider a second film to be stretched over a test hole of desired section in the same test plate with the above circular standard hole. The second film will have the same values of q and T as pertained for the circular standard film. An equation of equilibrium may be written for the test section in terms of its perimeter p , its area A , and the mean angle m around its boundary. The force acting upward will be qA . The force acting downward around the boundary will be $pT \sin m$. Equating

$$qA = pT \sin m$$

Eliminating t/q from the above two equations there remains

$$r = \frac{2A \sin \beta}{p \sin m}$$

If $r = 2A/p$, then $\beta = m$ and $q/T = (2/r) \sin \beta$

It is convenient to make the standard circular hole of the radius $2A/p$.

Appendix II

Saint Venant Analysis Applied to Bending.

Solution of bending problems shows that the distribution of shearing stresses over a cross section of the beam can be obtained by using a stress function satisfying equation: V

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = -\frac{\partial f(y)}{\partial y} + \frac{\mu}{1+\mu} \cdot \frac{Py}{I} + \frac{C}{\text{zero}} \quad (I)$$

and the boundary condition:

$$\frac{\partial \phi}{\partial s} = \left[\frac{Px^2}{2I} - f(y) \right] \frac{dy}{ds} \quad (II)$$

The stress components then are:

$$\tau_{xz} = \frac{\partial \phi}{\partial y} - \frac{Px^2}{2I} + f(y) \quad (III)$$

$$\tau_{yz} = -\frac{\partial \phi}{\partial x} \quad (IV)$$

By assuming that the bending load is so placed that twist of the beam is eliminated the constant C in equation (1) becomes zero.

Now adjust the function $f(y)$ in such a manner as to make the right side of equation (1) vanish. Then

$$\frac{\partial f(y)}{\partial y} = \frac{\mu}{1+\mu} \cdot \frac{Py}{I}$$

$$\text{Integrating} \quad f(y) = \frac{\mu}{1+\mu} \cdot \frac{Py^2}{2I}$$

Thus the problem reduces to the solution of the equation

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0 \quad (V)$$

with the boundary condition:

$$\frac{\partial \phi}{\partial s} = \left[\frac{Px^2}{2I} - \frac{\mu}{1+\mu} \cdot \frac{Py^2}{2I} \right] \frac{dy}{ds}$$

From this condition the values of function along the boundary are

$$\phi = \frac{P}{2I} \int x^2 dy - \frac{\mu}{2(1+\mu)} \cdot \frac{Py^3}{3} + \text{constant} \quad (VI)$$

^V See the paper by S. Timoshenko, Proc. London Math. Soc., 1921.



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It is seen that function ϕ is obtained from an unloaded soap film having along the boundary the ordinates calculated from equation (VI). This equation indicates that the boundary elevations vary along a smooth curve and come back to the starting point, because $\int x^2 dy$ is known to be zero around a closed area. The constant of equation (VI) need not be considered, as shearing stresses depend on the slopes $\frac{\partial \phi}{\partial y}$ and $-\frac{\partial \phi}{\partial x}$.

When the beam under consideration is constructed of steel, which permits that μ be taken as 0.3, equation (VI) becomes

$$\phi = \frac{P}{2I} \int x^2 dy - \frac{\mu^2 P}{26 I} \quad (\text{VII})$$

From equations (III) and (IV)

$$\tau_{xz} = \frac{\partial \phi}{\partial y} - \frac{Px^2}{2I} + \frac{\mu}{2 + 2\mu} \frac{Py^2}{I} \quad (\text{VIII})$$

$$\tau_{yz} = -\frac{\partial \phi}{\partial x} \quad (\text{IX})$$

If the material is steel with $\mu = .3$

$$\tau_{xz} = \frac{\partial \phi}{\partial y} - \frac{Px^2}{2I} + \frac{3}{26} \cdot \frac{Py^2}{I} \quad (\text{X})$$

$$\tau_{yz} = \frac{\partial \phi}{\partial x} \quad (\text{IX})$$

Equations (V, VII, X, and III) are the fundamental ones for the solution of problems by membrane analogy, when the material is steel.

It is necessary to take account of k and S as defined in equations (8, 9, 10 and 11).

By comparing equation (7) with (8) and (9), neglecting D

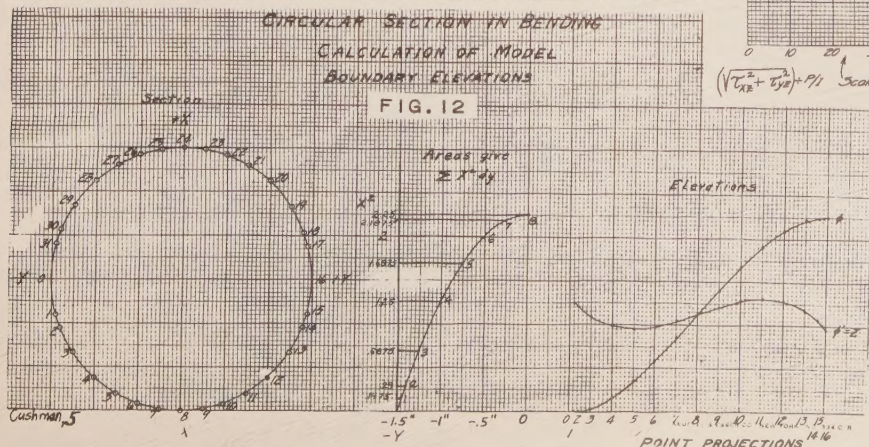
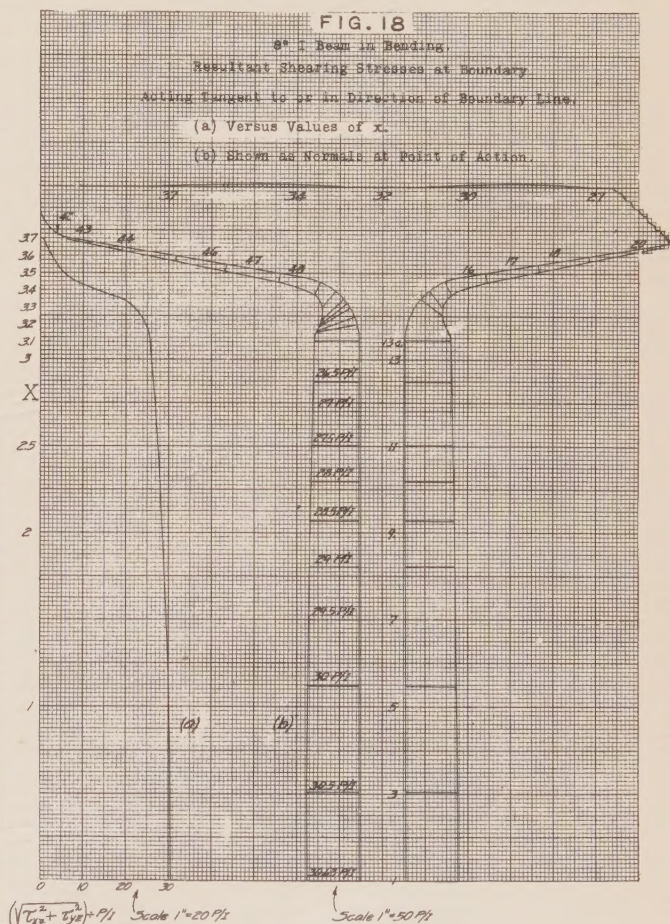
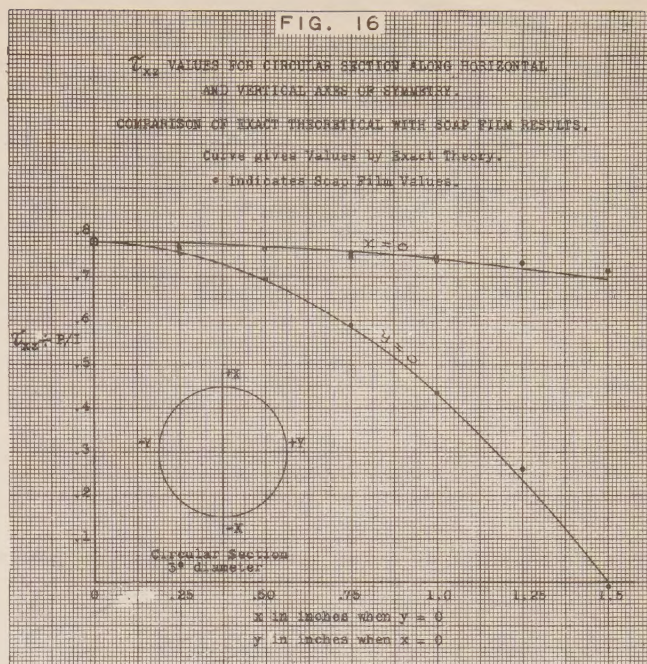
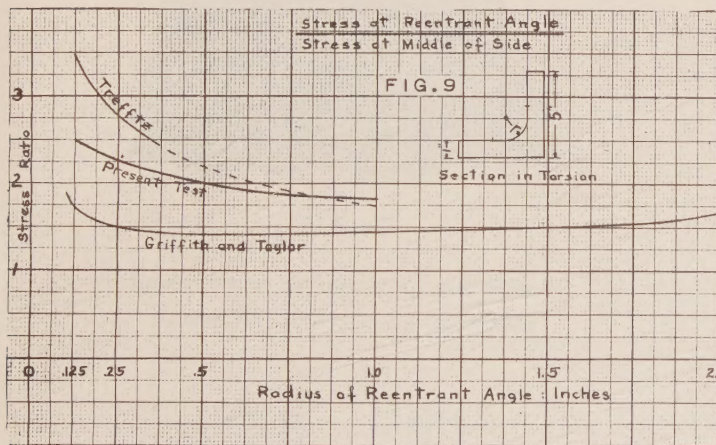
$$\phi = \frac{P}{I} (\phi' + Bx + Cy)$$

$$\phi = \frac{P}{I} (zk + Bx + Cy)$$

$$\frac{\partial \phi}{\partial y} = \frac{kP}{I} \frac{\partial z}{\partial y} + \frac{CP}{I} \quad (\text{XI})$$

$$-\frac{\partial \phi}{\partial x} = -\frac{kP}{I} \frac{\partial z}{\partial x} - \frac{BP}{I} \quad (\text{XII})$$

Substituting in equations (X) and (III), and taking account of the scale term S , there results equations (10) and (11) which are the more useful expressions for the shearing stresses in soap film analysis.



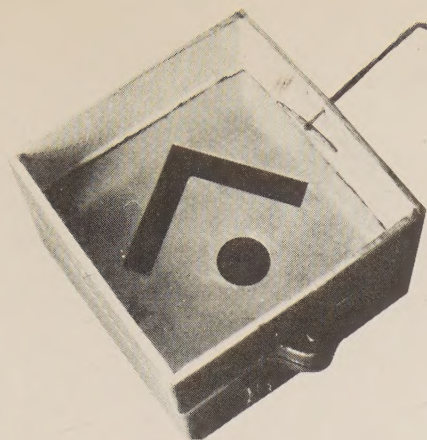


FIG. 3

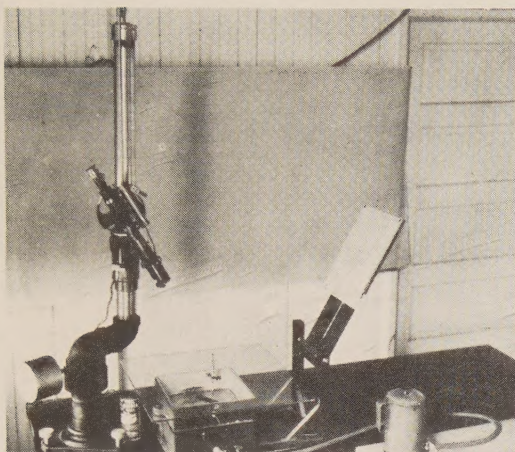


FIG. 7

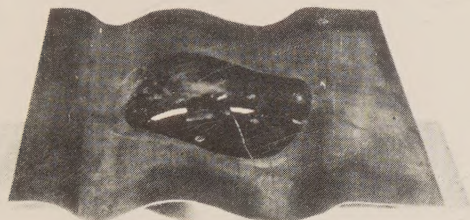


FIG. 11

SOAP FILM CONTOUR LINES FOR
CIRCULAR SECTION IN BENDING

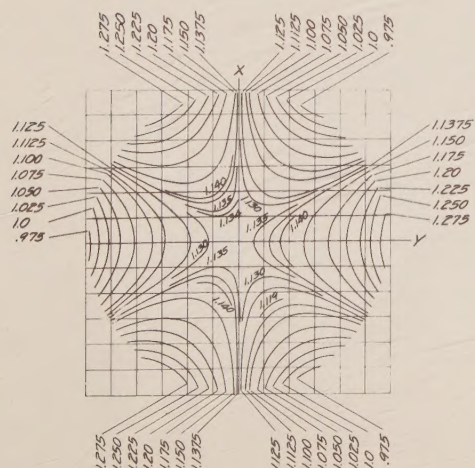


FIG. 15

SOAP FILM CONTOUR LINES FOR
5"x5"x1" ANGLE SECTION IN TORSION.
RADIUS AT REENTRANT ANGLE = .25"

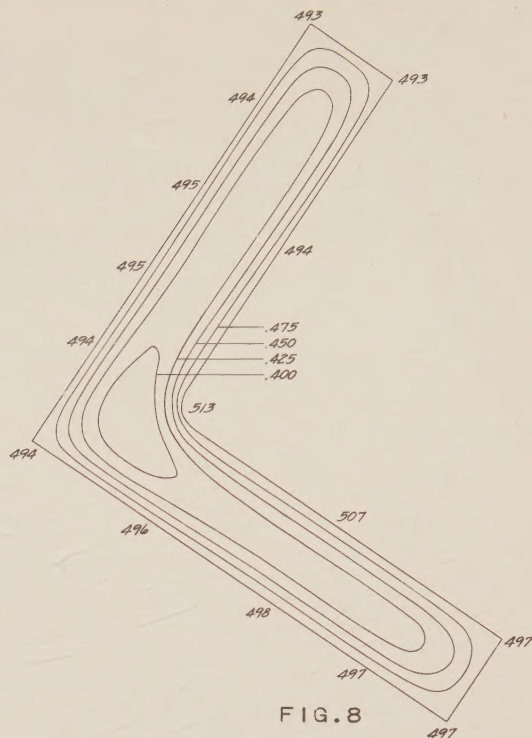


FIG. 8

SOAP FILM CONTOUR LINES FOR
8" I BEAM SECTION IN BENDING

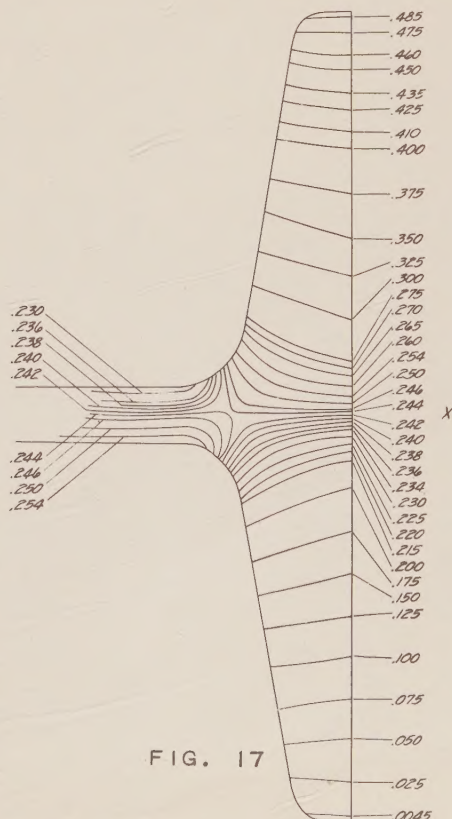


FIG. 17

